# Solutions

**Exam 2**
**Chapters 3 and 4**

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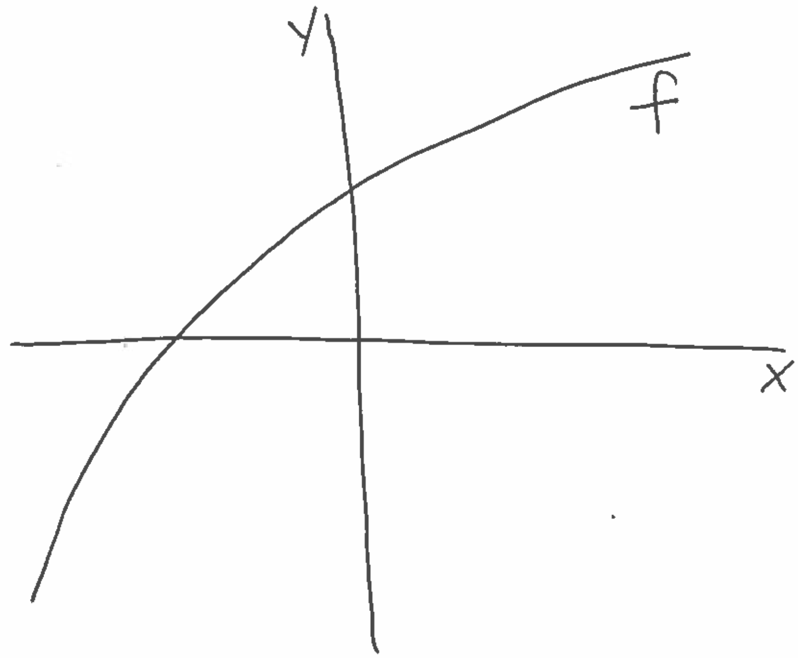
Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

**1.** (5 points) Consider the graph of the function $f$ given below. Determine the signs of its first and second derivative.

$f' > 0$

$f'' < 0$




**2.** (5 points) Find the derivative of the function $g(t) = e^t + 4t^5$.

$$\boxed{g'(t) = e^t + 20t^4}$$

**3.** (5 points) Find the derivative of $P = t^2 \ln t$.

Product Rule

$$\frac{dP}{dt} = 2t \cdot \ln t + t^2 \cdot \frac{1}{t}$$

$$\boxed{= t(2\ln t + 1)}$$

**4.** (5 points) Find the derivative of the function $f(x) = \frac{x^2}{1+e^{5x}}$.

Quotient Rule

$$\boxed{f'(x) = \frac{2x(1+e^{5x}) - x^2 \cdot 5e^{5x}}{(1+e^{5x})^2}}$$

**5.** (5 points) Find the derivative of $w = \ln(t^2)$.

$$\boxed{\frac{dw}{dt} = \frac{2}{t}}$$

Chain Rule

$w = \ln(u)$ where $u = t^2$

$$\frac{dw}{du} = \frac{1}{u} = \frac{1}{t^2} \qquad \frac{du}{dt} = 2t$$

**6.** (5 points) Find the derivative of the function $g(x) = te^{t^2}$.

$$g'(x) = e^{t^2} + t(e^{t^2})'$$

$$\boxed{= e^{t^2} + 2t^2 e^{t^2}}$$

Product Rule

Chain Rule

**7.** (10 points) Find all critical points and inflection points for the function $f(x) = 3x^5 - 5x^3$. For each of the critical points determine if it corresponds to a local maxima, local minima or neither. (No work=No credit)

Critical Points
$$\begin{cases} f'(x) = 15x^4 - 15x^2 = 15x^2(x^2-1) \\ f'(x) = 0 \text{ when } x = 0, \pm 1 \end{cases}$$

Inflection Points
$$\begin{cases} f''(x) = 60x^3 - 30x = 30x(2x^2-1) \\ f''(x) = 0 \text{ when } x = 0, \pm\frac{1}{\sqrt{2}} \end{cases}$$

Max and Min
$$\begin{cases} f''(0) = 0 & \text{so } x=0 \text{ is neither} \\ f''(-1) = -30 < 0 & \text{so } x=-1 \text{ is a local max} \\ f''(1) = 30 > 0 & \text{so } x=1 \text{ is a local min} \end{cases}$$

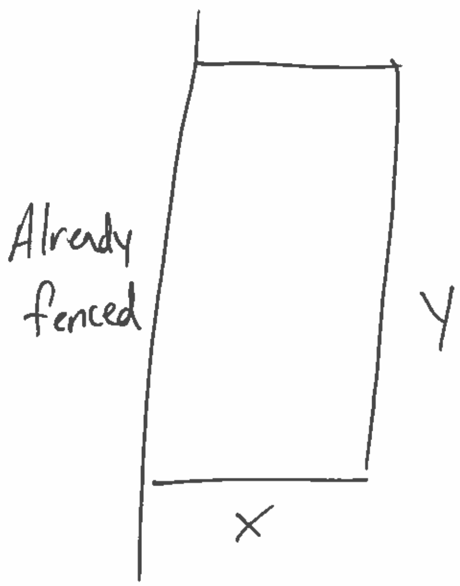
**8.** (10 points) Find the global maxima and minima for the function $g(x) = 2x^3 - 9x^2 + 12x + 1$ on the interval $0 \leq x \leq 3$.

$$g'(x) = 6x^2 - 18x + 12$$
$$= 6(x^2 - 3x + 2)$$
$$= 6(x-2)(x-1)$$

Critical Points at $x = 1, 2$

$\boxed{g(0) = 1 \text{ global min}}$ $g(1) = 6$ $g(2) = 5$ $\boxed{g(3) = 10 \text{ global Max}}$

**9.** (10 points) You have 1600 ft of fencing and you are trying to fence in a rectangular horse corral. However, your neighbor already has a fence along one side of your property which you will not need to fence off. What is the maximum area that you can enclose in your horse corral?




$$2x+y=1600 \Rightarrow y=1600-2x$$

$$\text{Area} = A = xy = x(1600-2x) = 1600x - 2x^2$$

$$\frac{dA}{dx} = 1600 - 4x$$

Critical point at $x = 400$

$\frac{d^2A}{dx^2} = -4$ so $x=400$ is a local max.

Thus $x = 400$ and $y = 800$ maximizes area.

Maximum Area $= 400 \cdot 800 = \boxed{320{,}000 \text{ square ft.}}$

**10.** (10 points) The demand equation for a quantity $q$ of a product at price $p$, in dollars, is $p = -5q + 5000$. Companies producing the product report the cost, $C$, in dollars, to produce a quantity $q$ is $C = 10q + 5$. For what quantity $q$ will profit be maximized? What is the maximum profit? (<u>Hint:</u> Revenue$=R = pq$.)

$$R = pq = (-5q + 5000)q = -5q^2 + 5000q$$

$$\text{Profit} = P = -5q^2 + 5000q - (10q + 5)$$

$$= -5q^2 + 4990q - 5$$

$$\frac{dP}{dq} = -10q + 4990; \text{ critical point when } \boxed{q = 499}.$$

$$\text{Maximum Profit} = P(499) = -5(499)^2 + 4990(499) - 5$$

$$\boxed{= \$1,245,000}$$

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**Bonus Question.** What makes a good friend?